The Randomness Delusion

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Abstract*. This paper proves that there is no true randomness in the natural world.*

Mathematics Subject Classification (2010). Primary 60A05; Secondary 62A01.

Keywords. True Randomness, Randomness Detector, PBSG-i_k, j-PBSG-i_k, BM.

Contents

1. Introduction

"– How can you know whether a sequence of numbers is random?"

The question above opens an interesting article about *Quantum Randomness*. [1] This my very little paper tries correctly theoretically to answer it: "– We cannot do it at all!"

This paper does not discuss the several ways of measuring statistical or computational characteristics from bits strings or binary sequences (or strings of symbols in general) associated to objective measures of randomness. [4] Otherwise, herein it is considered a very more fundamental question: whether we can really know whether an arbitrary binary sequence is or is not random; that is, whether true randomness really can exist in the natural world.

2. The Randomness Delusion

Definition 2.1: Schema of Periodical Binary Sequence Generator i_k (PBSG-i_k). Let **i** and **k** be non-negative integers such that $\mathbf{k} < \mathbf{2}^{\boldsymbol{i}}$. A *PBSG-i_k* is a DTM (Deterministic Turing Machine) [2] that ignores its input and generates (writes on its tape) continually [forever repeatedly] the **i**-sized binary sequence ***x*** that represents **k** (into an arbitrary 1-1 mapping $f : \{0, 1\}^{i} \rightarrow \{0, 1, 2, ..., 2^{i}\text{-}1\}$), that is, a periodical **k**-valued-sequence ***xxx...*** with period **i**, where $f(x) = \mathbf{k}$. (Obs.: PBSG-0_0 generates the null sequence.)

For instance, the PBSG-3_5 shall generate forever ***101101101...*** (Considering that *101* be mapped as the binary representation of 5, that is, $f(101) = \mathbf{5}$.)

Definition 2.2: True Randomness (Truly Random Sequence). An arbitrary sequence of bits has *true randomness* (or it is *truly random*) if its source is true random, i.e., there is neither deterministic source nor computation that can replace that source.

Definition 2.3: Barbosa Machine (BM). A BM is a machine like a DTM, but that decides correctly any decision problem, except when that problem is, directly or indirectly, about the behavior of the machine itself; hence, it detects and avoids anyway the self-referencing pitfalls.

Notice that a BM is an artificial, theoretical and very very powerful device, even though maybe it is impossible really to construct such a machine.

Definition 2.4: Truly Randomness Detector (TRD). Let a *TRD* be a DTM or BM that reads an arbitrary number of bits generated by any source and writes 1 on a specific cell into its tape if they are truly random (by Def. 2.2), or 0 otherwise, and then halts. That TRD computes anyway the [finite but unbounded] quantity of bits that it must read in order to attain its goal, and always eventually halts [within a finite time] with the correct answer. (Where the eventual randomness of that source cannot depend on the behavior of that TRD at all, since this behavior is deterministic: Hence, cannot exist self-referencing here at all.)

Definition 2.5: Schema of Initial Bits j from a PBSG-i_k (j-PBSG-i_k). Let **j** be a non-negative integer such that $\mathbf{j} \leq \mathbf{i}$. A *j-PBSG-i_k* is simply the sequence formed by the **j** initial bits from a PBSG-i_k. (Obs.: 0-PBSG-i_k is the null sequence.) For instance, 2-PBSG-3_5 = *10*.

Theorem 2.1. The Randomness Delusion. Any *TRD* will always write 0 on a specific tape cell and halt (meaning "–No, **w** is <u>not</u> truly random, it is <u>not</u> from a true random source.") on all the possible inputs **w**. Hence, does not exist such thing called *true randomness* at all, even into theoretical sense, since if it existed, then it would be detected some time, of course (imagine, for example, that a physicist say that the *dark energy* exists, but it is *absolutely undetectable*!...).

Proof. Let **w** be a |w|-sized binary sequence, and *M* be a DTM or BM intending be a TRD that on input **w** writes 1 on a specific tape cell and halts (meaning "– Yes, **w** is truly random, it is from a true random source."). All the PBSG-i_k whose |w|-PBSG-i_k = **w** will flatly deceive that TRD.

Note that these PBSG-i_k must exist, because |w| is arbitrary and unbounded, but finite, and **i** and **k** vary over all positive integers, covering all the possible finite sequences **w**, with the only constraining $\mathbf{k} < \mathbf{2}^{\boldsymbol{i}}$, where **w** is necessarily the representation of some **k**.

Since this DTM *M* is arbitrarily chosen and will be unavoidably wrong (detecting wrongly true randomness from, in fact, deterministic computation), then all the DTMs or BMs that answer "–Yes, **w** is truly random, it is from a true random source." on at least one input **w** cannot be a TRD at all. Hence the only answer from a real TRD on any input must be "– No, **w** is <u>not</u> truly random, it is <u>not</u> from a true random source." This fact logically implies that there is no true random source into universe, entailing the

randomness delusion (a source could be true random only if it should generate an infinite sequence of bits, but infinite objects do not exist in the real universe, evidently). □

3. Freedom & Mathematics

"**– The essence of Mathematics is Freedom.**" (Georg Cantor) [3]

4. References

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[3] From The Engines of Our Ingenuity, site, "*Episode nº 1484: GEORG CANTOR*", posted by John H. Lienhard, unpublished, available: http://www.uh.edu/engines/epi1484.htm

[4] R. G. Downey and D. R. Hirschfeldt, *Algorithmic Randomness and Complexity*, Springer-Verlag, New York NY, 2010.

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